Division **E**

# Mathematical Olympiads

**FEBRUARY 7, 2006**

*for Elementary and Middle Schools*

Contest **4**

**4A** ***Time: 4 minutes***

Five students (Amy, Beth, Corey, Diego, Emily) sit in that order in a circle, counting down to 1. Amy starts by saying, "34". Then Beth says, "33", and so on. They continue around the circle to count down by ones. Who says, "1"?

**4B** ***Time: 5 minutes***

What whole number may be used in place of □ to make this statement true?

$$\frac{3}{5} < \frac{\square}{7} < \frac{4}{5}$$

**4C** ***Time: 6 minutes***

Bay Street has between 1 and 15 houses, numbered 1, 2, 3, and so on. Mr. Sullivan lives in one of the houses. The sum of all the house numbers less than his equals the sum of all the house numbers greater than his. How many houses are there on Bay Street?

**4D** ***Time: 6 minutes***

Eight cubes are glued together to form the figure shown. The length of an edge of each cube is 3 cm. The entire figure is covered in paint. How many square centimeters are covered in paint?

**4E** ***Time: 7 minutes***

The whole number $N$ is divisible by 7. $N$ leaves a remainder of 1 when divided by 2, 3, 4, or 5. What is the smallest value that $N$ can be?

*Please fold over on line. Write answers on back.*

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Division **E**

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Contest **4**

**4A** *Student Name and Answer*

**4B** *Student Name and Answer*

**4C** *Student Name and Answer*

houses

**4D** *Student Name and Answer*

sq cm

**4E** *Student Name and Answer*

*Please fold over on line. Write answers in these boxes.*

Division E

Mathematical Olympiads
FEBRUARY 7, 2006
for Elementary and Middle Schools

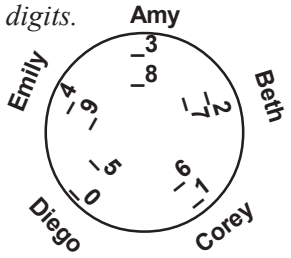
Contest 4

# SOLUTIONS AND ANSWERS

*Items in parentheses are not required.*

**4A** **METHOD 1:** *Strategy: Count up.*
Since there are five people in the circle, the person who says "1" will also say 6, 11, 16, 21, 26, and 31. If Amy says 34, and Beth says 33, then Corey says 32, and Diego says 31. Therefore, **the person who says "1" is Diego.**

**METHOD 2:** *Strategy: Count down, noticing the units digits.*
There are five students, so after counting twice around the table, they have counted down ten numbers. After that, the students count the same units digits as they did the first two rounds. The student who says "1" is the same student who says "31", Diego.




***FOLLOW-UP:*** *Suppose the numbers 1 through 100 alternate direction in every other row as indicated in the table at the right. In which column does 49 appear? 100?*[A, D]

| A | B | C | D | E | F |
|---|---|---|---|---|---|
| 1 | 2 | 3 | 4 | 5 | 6 |
| 12 | 11 | 10 | 9 | 8 | 7 |
| 13 | 14 | 15 | … | | |

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**4B** *Strategy: Find a common denominator.*
The least common denominator is 35, which is the least common multiple of 5 and 7. Raising the terms, the statement becomes $\frac{21}{35} < \frac{5\times\square}{35} < \frac{28}{35}$. The numerator of the middle fraction is a multiple of 5. The only multiple of 5 between 21 and 28 is 25. If 5 × ☐ is 25, **the whole number used for ☐ is 5.**

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**4C** *Strategy: Make a table.*
Consider the first few houses on a street, then the house Mr. Sullivan might live in, and then the first few houses that come after that.

| Beginning of Street | Mr. Sullivan | End of Street |
|---|---|---|
| 1 + 2 + 3 = **6** | 4 | 5 + 6 = **11** ..... ***Too big*** |
| 1 + 2 + 3 + 4 = **10** | 5 | 6 + 7 = **13** ..... ***Too big*** |
| 1 + 2 + 3 + 4 + 5 = **15** | 6 | 7 + 8 = **15** ..... ***Equal sums*** |

Therefore, **there are 8 houses on Bay Street.**

***FOLLOW-UP:*** *Suppose the product of the house numbers before Mr. Sullivan's is the same as that of the house numbers after his. How many houses are on Bay Street?* [10]

**4A** **Diego**

**4B** *(☐ is)* **5**

**4C** **8** *(houses)*

**4D** **288** *(sq cm)*

**4E** **301**

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**4D** **METHOD 1:** *Strategy: Count the number of cube faces that are exposed.*

The front of the figure has 8 cube faces and the back has 8 faces. The top, bottom, and two sides each have 3 faces. The middle "hole" has 4 exposed faces. In all, there are 8 + 8 + 3 + 3 + 3 + 3 + 4 or 32 exposed faces that are painted. The area of each face is 3 × 3 or 9 sq cm, so 32 × 9 or **288 sq cm are covered in paint.**

**METHOD 2:** *Strategy: Count the number of cube faces that are not exposed.*
Each cube has 6 faces and there are 8 cubes in the figure for a total of 48 faces. There are 8 places where 2 cube faces are glued together and so 8 × 2 faces that are not painted. Then 48 – 16 or 32 faces are painted. As above, 288 sq cm are covered in paint.

**4E** **METHOD 1:** *Strategy: Use the least common multiple.*
*N* leaves a remainder of 1 when divided by 2, 3, 4, or 5. Suppose we subtract 1 from *N*. The result is a multiple of 2, 3, 4, and 5. The least common multiple of all four numbers is 60. Moreover, *all* common multiples of 2, 3, 4, and 5 are multiples of 60. Then *N* is 1 more than a multiple of 60. *N* is in the set {61, 121, 181, 241, 301, 361, …}. Divide each of these by 7. The smallest of them that is a multiple of 7 is 301. **The smallest value that N can be is 301.**

**METHOD 2:** *Strategy: Determine the units digit and then the possible multiples of 7.*
*N* leaves a remainder of 1 when divided by 5, so *N* has a units digit of 1 or 6. *N* leaves a remainder of 1 when divided by 2, so *N* is odd. Therefore the units digit is 1. The multiples of 7 that have a units digit of 1 are the product of 7 and a number with a units digit of 3; i.e. 7 × 3, 7 × 13, etc. Then *N* is one of the numbers in the set {21, 91, 161, 231, 301,371, …}. The smallest of these that leaves a remainder of 1 when divided by 3 or 4 is 301.

***Follow-Ups:*** *(1) A class has more than 10 students. The teacher tries to group them for a game. If she forms groups of 3, 4, 6, or 8, one student is left out. How many students are in the class?* [25] *(2) What is the smallest number that leaves a remainder of 1 when divided by 2, a remainder of 2 when divided by 3, a remainder of 3 when divided by 4, a remainder of 4 when divided by 5, a remainder of 5 when divided by 6, and a remainder of 6 when divided by 7?*[419; Hint: What happens if 1 is added to the number?]

***NOTE:*** *Other problems related to some of the above can be found in our books "Math Olympiad Contest Problems for Elementary and Middle Schools" and "Creative Problem Solving in School Mathematics."*